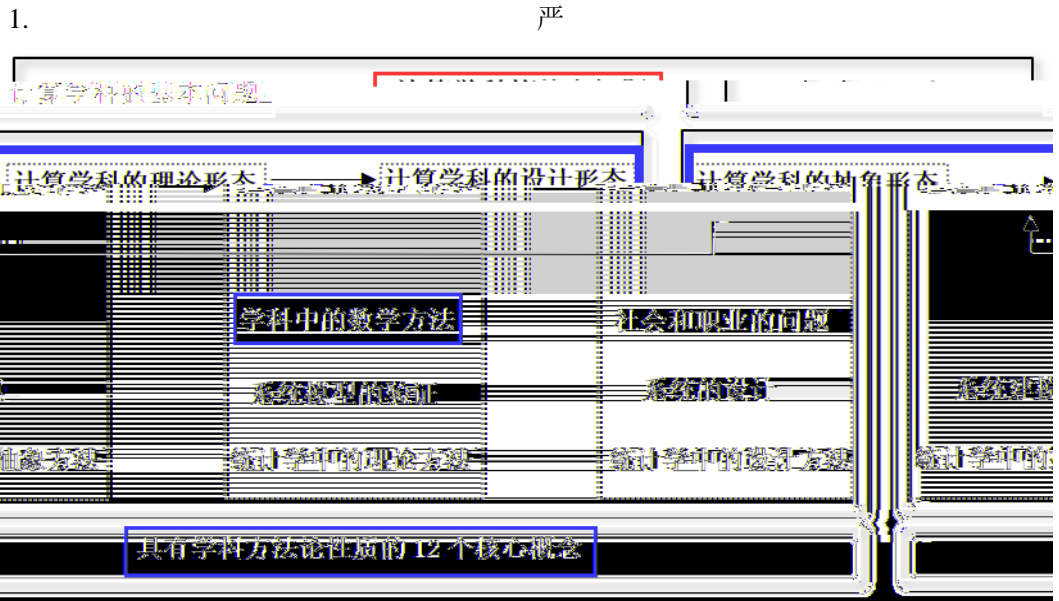


# 科学思维-样例：RSA 公开密钥密码系统

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1976

Whitfield Diffie

Martin Hellman

Diffie Hellman key exchange DH

1976

*New Directions in Cryptography*

1976

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1978

R. L. Rivest

A. Shamir

L. M. Adleman

*A Method for Obtaining Digital Signatures and Public-Key Cryptosystems*

1978

RSA

RSA

RSA

2002

RSA= $\langle p, q, n, m, e, d, k, c \rangle$

1978

1  $p, q, n, m, e, d, k, c \in Z^*, Z^* = \{1, 2, 3, \dots\}$

2  $p, q$  各  $n = p \times q$

3  $(e, n): (d, n):$

4  $m: m < n$

5  $c:$

6  $k(m^{k(p-1)(q-1)} \pmod n) = 1$

7  $k(ed = k(p-1)(q-1)+1)$

8  $c = m^e \pmod n$

9  $m = c^d \pmod n$

1 各  $p, q$

2  $e \in (p-1)(q-1) \quad 0 < e < (p-1)(q-1)$

3  $d \in k(ed = k(p-1)(q-1)+1)$

3. RSA 1978

1  $m \quad c = m^e \pmod n$

2  $c \quad m = c^d \pmod n$

RSA 1978  $(e, n) \quad (d, n) \quad p \quad q$

$k(m^{k(p-1)(q-1)} \pmod n) = 1$

CS

例  $p=3, q=11, n = 3 \times 11 = 33$

$m=2 \quad m < n, k=1$

$m^{k(p-1)(q-1)} \pmod n = 2^{1 \times (3-1) \times (11-1)} \pmod 33$

$= 2^{20} \pmod 33$

$= 1048576 \pmod 33$

$$\begin{aligned}
&= 1 \\
& \quad m=2 \quad m < n \quad , k=2 \\
m^{k(p-1)(q-1)}(\bmod n) &= 2^{2 \times (3-1) \times (11-1)}(\bmod 33) \\
&= 2^{40}(\bmod 33) \\
&= 1\,099\,511\,627\,776 \pmod{33} \\
&= 1 \\
& \quad m=2 \quad m < n \quad , k=3 \\
m^{k(p-1)(q-1)}(\bmod n) &= 2^{3 \times (3-1) \times (11-1)}(\bmod
\end{aligned}$$

$$=2187 \pmod{33}$$

$$=9$$

例  $p=223092827, q=218610473 \quad n=487\,704\,284\,333\,771\,171$  RSA

$p, q, n$

$e$

$$p=223\,092\,827, q=218\,610\,473 \quad (p-1) \times (q-1) = (223\,092\,827-1) \times (218\,610\,473-1) \\ = 48\,770\,427\,991\,673\,872$$

RSA

$$e \quad 48\,770\,427\,991\,673\,872$$

$$e=2 \quad 48\,770\,427\,991$$

⇒

$p=11, q=13$

RSA

9